

Performance Analysis of Channel Estimation Schemes for Phase Shift Optimization: An Analysis of Bit Error Rate and Armijo Step Size Behavior

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Abstract— In the recent past, there has been a growing need for ultra-low latency and high-data-rate communication. In Non-Line-of-Sight (NLoS) communication, the channel capacity and accuracy of transmission are significantly affected by interferences, lowering the Quality of Service (QoS). An intelligent Reflecting Surface (IRS) has risen as a potential solution to challenges associated with NLoS communication including low data rate, multipath fading, and high BER. However, to leverage the performance gains of the IRS, effective and highly accurate channel estimation is crucial as it facilitates optimal phase shift optimization. This work investigated the performance of four main channel estimation algorithms in an IRS-aided system; LS, DD, DFT, and MMSE in terms of their BERs and effects on the convergence behavior of the Stochastic Convex Approximation (SCA) algorithm following the Armijo rule. The objectives of this work were to determine how different channel estimation schemes influence the BER and test the different rates of convergence. Results indicate that in cases without statistical knowledge of the channel, the DD method provides the best performance. The main advantage of the DD method is that it effectively tracks the possibly varying channels and provides an effective update technique that is not dependent on pilot symbols. This work shows that the communication needs, complexity, and accuracy should be carefully considered when selecting the channel estimation method for IRS-aided communication systems. The outcomes of this research have a critical role in shaping future wireless communication systems by aiding in the adoption of the most optimal channel estimation schemes that fit with specific user needs and resource constraints.

Keywords: IRS, Communication, LS, DD, MMSE, DFT, BER, Armijo

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1. Introduction

In today's wireless communication systems, there is a greater need for high speed, low latency, and high accuracy transmission to meet the rising demands posed by data-hungry IoT devices and communication systems [1]. An Intelligent Reflecting Surface (IRS) has gained popularity in the research community as one of the potential solutions to the challenges that plague the wireless environment. According to [2], these challenges include multipath fading, interference, and high path loss, which are common in Non-Line-Of-Sight (NLoS) systems. In such conditions, accurate channel estimation and phase shift optimization are vital. Channel estimation entails estimating the Channel Impulse Response (CIR), for optimized transmission, which leads to an increase in the data throughput, minimal Bit Error Rate (BER), and enhanced spectral efficiency [3].

The Least Squares (LS), Decision-Directed (DD), Minimum Mean Square Error (MMSE) and

Discrete Fourier Transform (DFT) are some of the commonly used channel estimation methods in IRS-aided systems [4]. The LS estimation method is a pilot-based technique characterized by simplicity and computational efficiency, which have made it common in IRS-aided systems [5], [6], [7], [8]. Further, the LS method was applied in [9] with an ON/OFF IRS mode and shown to offer fairly accurate results with minimal complexity. However, it does not utilize the statistical properties of the channel and can be prone to performance decline in low Signal-to-Noise Ratio (SNR) settings.

The DD estimation scheme, on the other hand, is based on initial estimates from the pilot symbols and then uses the decoded data symbols for subsequent channel tracking and update [10]. The Decision-Directed (DD) method has been found to enhance data accuracy as it has the lowest BER [11] and an order of magnitude lower complexity compared to machine-learning channel estimation algorithms [12]. Further, its application in WLAN systems has been investigated, showing that it leads to low PER and can compensate for channel distortions as indicated in [13] - [14].

The MMSE estimation methodology is a combination of LS estimate and the statistical knowledge of the channel and noise. MMSE leads to a better performance than LS and DD estimation, but it has the disadvantage of a higher computational complexity and requires knowledge of the channel statistics [15]. Authors in [16] tested the performance of MMSE in time-varying environments and proved its robustness and applicability in LTE systems.

On the other hand, the DFT estimation method utilizes the underlying structure of the channel in the frequency domain, which makes it very important for orthogonal frequency-division multiplexing (OFDM) systems [17]. Authors in [18] designed an IRS activation pattern following a series of DFTs and reported that in comparison to the LS method, the DFT approach has a variance of one order less, implying that it significantly reduces the training period. However, DFT estimators have been reported to experience a high error floor at high SNRs [19].

An IRS is a diffuse scatterer whose phase shifts must be optimized for spectral efficiency and enhanced data rates. Optimization entails determining the best operating conditions to achieve the desired output. In this case, the IRS optimization process requires a determination of optimal phase shifts and/or amplitude shifts. Some of the phase shift optimization methods used include Semidefinite relaxation and stochastic convex approximation (SCA). From the research done by [20], the SCA algorithm outperforms the semidefinite relaxation method in terms of complexity and convergence rate.

The fundamental concept underlying SCA algorithm is to iteratively approximate the non-convex objective functions and constraints with convex approximations, which can then be solved effectively using normal convex optimization methods [21]. The SCA method relies on gradient descent, in which the Armijo rule finds application. The Armijo rule is a well-known backtracking line search technique used in gradient descent algorithms for the optimization of phase shifts in IRS systems aimed at maximizing the Signal-to-Interference-Plus- Noise- Ratio (SINR) to improve the overall system performance [22]. Channel estimates provide a better initialization for the Armijo rule which follows gradient descent [23].

The primary objectives of this research were twofold; comparing the BER performance of the LS, DD, MMSE, and DFT channel estimation schemes under different channel conditions and analyzing their effect on the convergence property of the Stochastic Convex Approximation (SCA) algorithm for phase shift optimization. Through the performance and convergence assessment of the SCA algorithm following the Armijo rule with different channel estimation schemes, this research highlights the available trade-offs that should be taken into account while selecting a fitting channel estimation technique for optimal passive beamforming in IRS-aided systems.

2. Method

2.1 Simulation Set-up and Channel Modelling

This work focused on an NLOS WLAN system with the direct path from the Access Point (AP) to the User Equipment (UE) being completely blocked. An IRS with 100 passive elements was introduced midway between the user and the AP to provide a virtual Line-Of-Sight LOS for the signals. This central positioning is based on the need to mitigate the multiplicative path loss effect associated with IRS systems which increases with an increase in distance [24]. A pictorial representation of the system is shown in Figure 1 below.

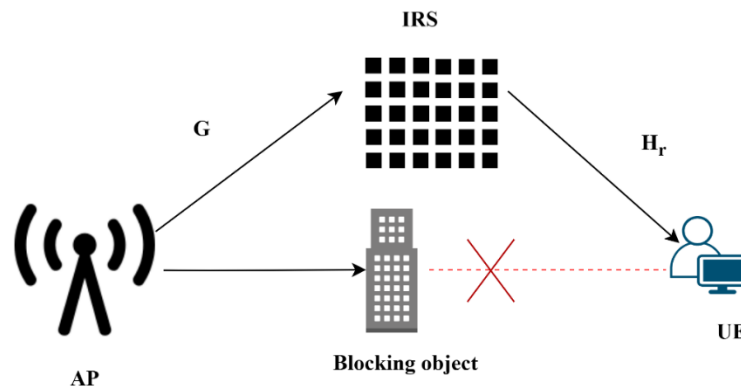


Figure 1. NLOS IRS-aided system

A Single Input Single Output (SISO) system with the AP and UE each having one receive antenna. A Uniform Rectangular Array (URA) is considered for the IRS elements which are separated by $\lambda/2$. This separation ensures no inter-element interference [25]. The IRS is in the Fraunhofer far-field of the AP and UE and, therefore, the IRS channels can be modelled using a simple geometric model with the IRS being treated as a single reflector with no mutual coupling [26] - [27]. Channels G and H_r are modelled as;

$$G = P_g \left(\sqrt{\frac{\varepsilon}{\varepsilon + 1}} \right) a_N(\vartheta) a_M(\varphi)^H + \sqrt{\frac{1}{\varepsilon + 1}} \tilde{G} \quad (1)$$

$$H_r = P_s \left(\sqrt{\frac{\varepsilon}{\varepsilon + 1}} \right) a_N(\zeta) + \sqrt{\frac{1}{\varepsilon + 1}} \tilde{H}_r \quad (2)$$

Where;

$G \in \mathbb{C}^{N \times 1}$ - is the AP-IRS channel

$H_r \in \mathbb{C}^{N \times 1}$ - is the IRS-UE channel

ε - is the Rician factor

\tilde{G} and \tilde{H}_r are NLOS components whose elements are chosen from $\mathcal{CN}(0,1)$

α is the steering vector and ϑ , φ , and ζ are the angular parameters

P_g and P_s are the path losses for channels G and Hr, and are modelled following the IEEE 802.11ax standard as in equation (3) below.

$$PL_{outdoor-LOS}(d) = 22.0 \log_{10}(d) + 28 + 20 \log_{10}(f_c) \tag{3}$$

2.2 Channel Estimation

For a completely passive IRS, it is more cost-effective to estimate the cascaded channel $H_r \phi G$. To achieve this, block-type pilots are transmitted from the UE to the AP and used to estimate the cascaded channel gain. This work considered a constant amplitude, continuous phase shift approach where the phase of reflecting element n is given as $\theta_n = e^{j\varphi}$ for $\varphi \in [0, 2\pi]$ which ensures maximum reflection at an amplitude of 1 [28]. For the transmission protocol, a Time Division Duplexing (TDD) protocol is considered for uplink and downlink transmissions and channel reciprocity for the CSI acquisition in the downlink based on the uplink training is considered as shown in Figure 2 below.

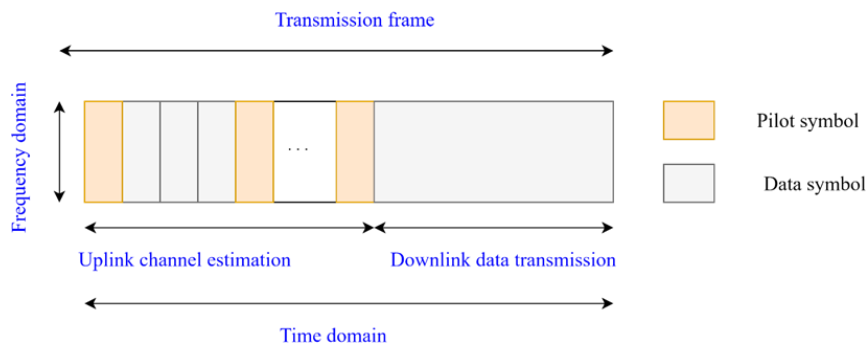


Figure 2. The TDD transmission protocol

Let the UE transmit X data symbols, the signal received at the AP is given as;

$$Y = H_r^H \phi G X + Z \tag{4}$$

Where;

Z -noise at the AP

Equation (4) can be rewritten as;

$$Y = H_{cascaded} X + Z \tag{5}$$

For a total of K subcarriers, the received training symbol following OFDM transmission is given by equation (6);

$$Y \triangleq \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{[K-1]} \end{bmatrix} = \begin{bmatrix} X_0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & X_1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & X_{[K-1]} \end{bmatrix} \begin{bmatrix} H_0 \\ H_1 \\ \cdot \\ \cdot \\ \cdot \\ H_{[K-1]} \end{bmatrix} + \begin{bmatrix} Z_0 \\ Z_1 \\ \cdot \\ \cdot \\ \cdot \\ Z_{[K-1]} \end{bmatrix} \quad (6)$$

$H = [H_0, H_1, \dots, H_{[K-1]}]^T$ is the channel vector representing $H_{cascaded}$ in equation (5) above.

2.2.1 LS Channel Estimation

The LS estimate obtained from the extracted pilot symbols is given as;

$$\tilde{H}_{LS,k} = \frac{Y_k}{X_k} \quad \forall k = 0, 1, 2 \dots K - 1 \quad (7)$$

2.2.2 DD Channel Estimation

DD channel estimation is done to update the channel estimates using decoded symbols. The DD estimation process is represented in the algorithm below, where the data symbols need to be extracted, compensated by the channel estimates for the previous symbols and updated using the compensated symbols.

DD Estimation Algorithm

1 input:

The L^{th} received OFDM symbol, $\hat{x}_L[k]$

2 Retrieve the Channel estimate for the $L-1^{\text{th}}$ estimate from the LS estimates, $\tilde{H}_{L-1}[k]$

3 Compensate for the L^{th} received symbol using the $L-1^{\text{th}}$ estimate, $\hat{x}_L[k] = \frac{Y_L[k]}{\tilde{H}_{L-1}[k]}$

4 Make hard decisions for the compensated symbols, $\bar{x}_L[k]$

5 Update the channel estimate by the compensated symbols, $\tilde{H}_{DD}[k] = \frac{Y_L[k]}{\bar{x}_L[k]}$

2.2.3 MMSE Channel Estimation

The primary goal in MMSE is to minimize the Mean Square Error (MSE) by finding a better linear estimate. Using the LS estimates, the MMSE estimate is given as;

$$\tilde{H}_{MMSE} = R_{H\tilde{H}} R_{\tilde{H}\tilde{H}}^{-1} \tilde{H} \quad (8)$$

Where;

$R_{H\tilde{H}}$ – is the cross – correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain.

$R_{\tilde{H}\tilde{H}}$ – is the autocorrelation matrix of \tilde{H}

\tilde{H} – is the LS channel estimate

2.2.4 DFT Channel Estimation

The DFT-based channel estimation method improves the performance of LS or MMSE channel estimation by eliminating the effect of noise outside the maximum channel delay thus minimizing the effect of noise as detailed in [29]. Taking the IDFT of the LS estimate, the DFT channel estimate for a maximum channel delay L , is given as;

$$\hat{h}_{DFT}[n] = IDFT\{\tilde{H}_{LS,k}\} = h[n] + z[n] \quad \forall n = 0, 1, 2, \dots, L - 1 \quad (9)$$

Transforming back to the frequency domain we have;

$$\tilde{H}_{DFT}[k] = DFT\{\hat{h}_{DFT}[n]\} \quad (10)$$

In summary, the process for BER computation is illustrated in the flowchart below;

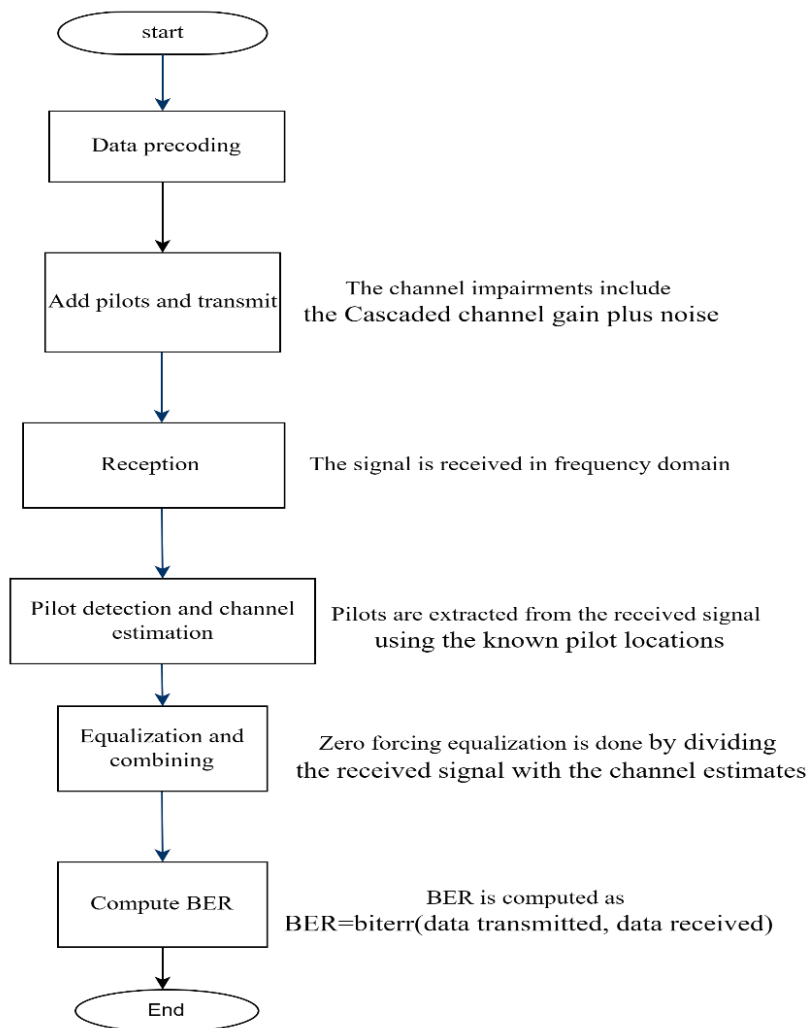


Figure 3. BER computation flowchart

The following section describes how the performance of these four channel estimation methods was examined as they were used to initialize the Armijo algorithm, showing how they influence the convergence behavior and step sizes.

2.3 Optimization

In this step, the goal was to find the optimal θ that maximizes the achievable rate. From equation (4), the SINR is shown in equation (11) as;

$$\gamma = \frac{|(H_r^H \theta G)w|}{|(H_{r,u}^H \theta G)w_j| + \sigma^2} \quad (11)$$

With the beamforming weight w designed as matched beamforming using the channel estimates, the optimization problem is formulated as;

$$P(1) \max_{\theta} f_1(\theta) = w \log(1 + \gamma) \quad (12)$$

θ is updated by;

$$\theta = \underset{\theta}{\operatorname{argmin}} 2(\theta) \triangleq \theta^H U \theta - 2 \operatorname{Re}\{\theta^H v\} \quad (11)$$

Where;

$$U = |\bar{\beta}|^2 \bar{a} \bar{a}^H \quad (12)$$

$$v = (\sqrt{w(1 + \bar{\alpha})} \bar{\beta}^* \bar{a} - |\beta_u|^2 \bar{a}) \quad (13)$$

Where;

α is the effective channel gain $H_{cascaded} w$

β and α are auxiliary variables

Further, since $\theta_n = e^{i\varphi_n}$ and $\varphi_n \in \mathbb{R}$, the update rule is recast to;

$$\varphi = \underset{\varphi \in \mathbb{R}^N}{\operatorname{argmin}} f_2(\varphi) \triangleq (e^{i\varphi})^T U e^{i\varphi} - 2 \operatorname{Re}\{v^H e^{i\varphi}\} \quad (14)$$

Where $\varphi = [\varphi_1, \dots, \varphi_N]^T$

f_2 is non-convex and, therefore, a surrogate function is formulated using SCA which is solved to yield a stationary solution for f_2 . This gives $\varphi = \underset{\varphi \in \mathbb{R}^N}{\operatorname{argmin}} f_3(\varphi, \bar{\varphi})$ which must satisfy two conditions;

$$f_3(\bar{\varphi}, \bar{\varphi}) = f_2(\bar{\varphi}) \quad (15)$$

$$f_3(\varphi, \bar{\varphi}) \geq f_2(\varphi)$$

Using Taylor expansion, f_3 can be formulated as;

$$f_3(\bar{\varphi}, \bar{\varphi}) \geq f_2(\bar{\varphi}) + \nabla f_2(\bar{\varphi})^T((\varphi - \bar{\varphi}) + \frac{\kappa}{2} \|\varphi - \bar{\varphi}\|^2) \quad (16)$$

Thus, φ is updated by;

$$\varphi = \bar{\varphi} - \frac{\nabla f_2(\bar{\varphi})}{\kappa} \quad (17)$$

Where κ is chosen by Armijo rule. For a total of R iterations, the phase shift is updated by;

$$\varphi = \varphi_r = \bar{\varphi}_r - \frac{\nabla f_2(\bar{\varphi}_r)}{\kappa_r} \quad \forall r = 1, 2, \dots, R \quad (18)$$

The solution guarantees a stationary solution for P (1) which is the optimal solution for the data rate maximization problem. For each simulation point, an average of 10^2 iterations were performed. The algorithm is summarized below;

Algorithm: SCA Update for θ

- 1 Input:
 Computed channel estimates
- 2 Set $r=0$ and compute the beamforming weight with the channel estimates
- 3 Initialize φ to feasible values
- 4 Set $r=r+1$
- 5 Compute U and v by (12) and (13)
- 6 Search κ by Armijo rule
- 7 Update φ by (18)
- 8 Repeat steps 3-7 until the value of f_3 converges

3. Result and Discussion

This section presents a discussion of the results obtained from this research. It should be noted that in this case, a quasi-static model was considered, where the AP, IRS and UE are in fixed positions. BER is a crucial factor in communication systems as it determines the accuracy of data transmitted, particularly in NLOS scenarios where interferences can cause significant distortion in the transmitted data. The main goal of this work was to examine how various channel estimation schemes affect the accuracy and capacity by investigating BER and the convergence behavior of the phase shift algorithm. The outcomes are presented below.

3.1 Error Rates

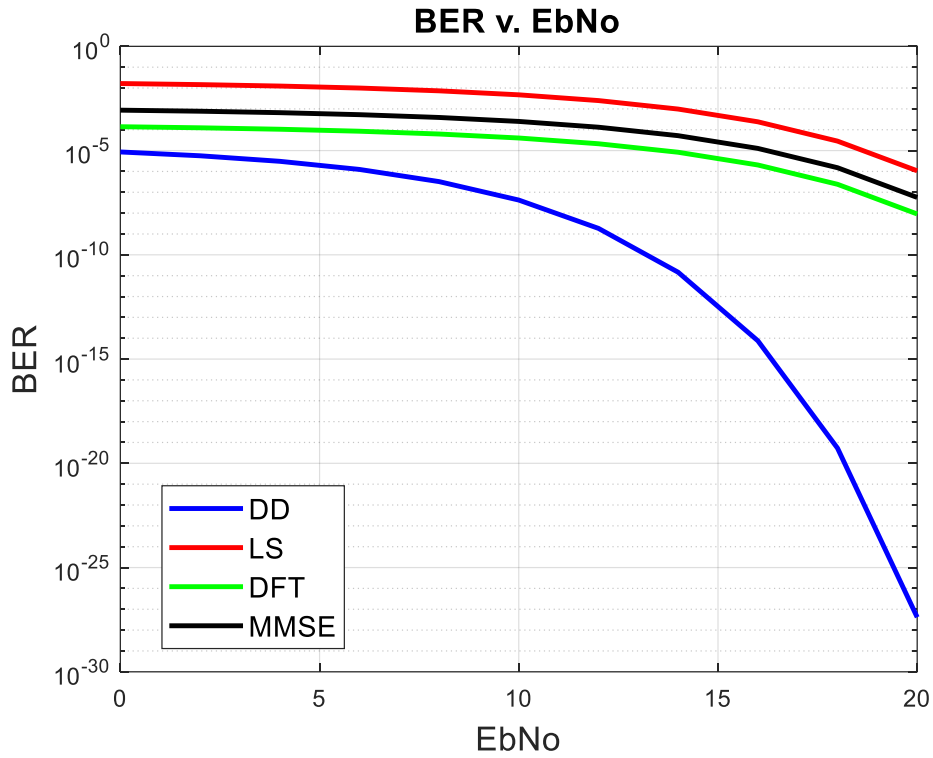


Figure 4. BER v. E_b/N_0

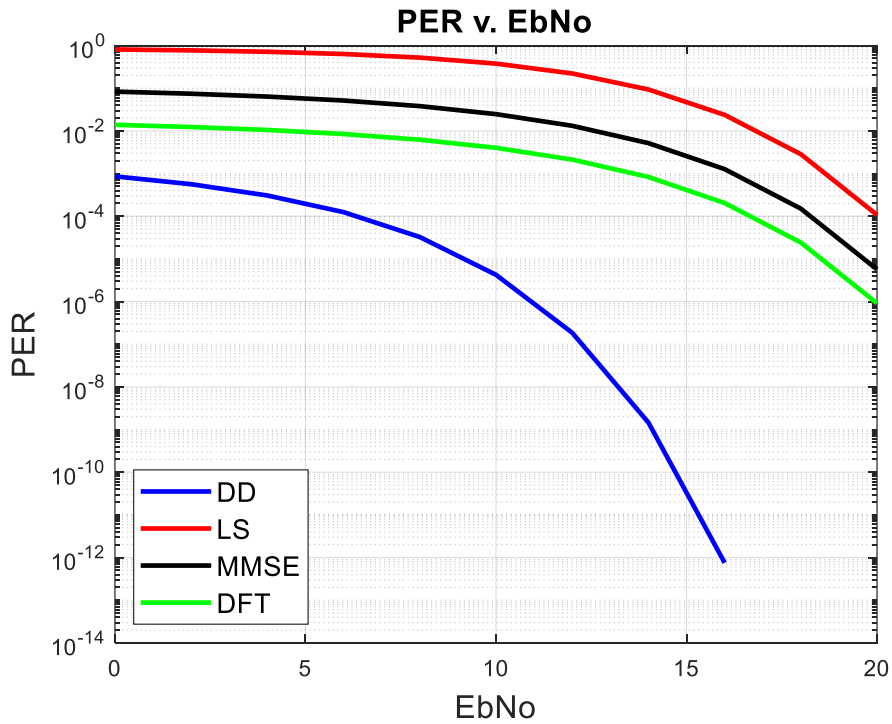


Figure 5. PER v. E_b/N_0

The DD channel estimation scheme exhibits the best BER and PER performance among the four methods across the entire E_b/N_0 range as it has the lowest error values, indicating its superiority in minimizing bit errors. This superior performance can be attributed to the channel tracking and update

capabilities of the DD method. In regards to complexity, the DD approach is independent of the IRS as it does not require subsequent transmission of pilots after the data symbols are decoded, while the LS requires N training symbols to estimate the channel coefficients associated with the IRS [30]. This implies that DD has a lower complexity than the LS, MMSE, and DFT methods as they depend on pilot transmission.

The DFT estimation scheme shows the second-best BER and PER performance after DD. At lower EbNo values, the DFT curve closely follows the DD curve, suggesting comparable performance in low SNR conditions. However, as the EbNo increases, the gap between DD and DFT widens, indicating that the DD scheme can better exploit the higher SNR to further reduce the BER than the DFT approach.

The MMSE estimation scheme's BER performance falls between the DFT and LS methods. At lower EbNo values, the MMSE curve closely follows the LS curve, indicating similar performance in low SNR conditions. However, as the EbNo increases, the MMSE curve gradually approaches the DFT curve, suggesting that it can exploit the higher SNR to improve its BER performance compared to LS. This improved performance can be attributed to the MMSE technique's optimal estimation of the channel by leveraging the statistical knowledge of the channel and noise.

The LS estimation method exhibits the worst BER performance among the four schemes across the entire EbNo range. The LS curve consistently has higher BER values compared to the other methods. This can be attributed to the fact that LS estimation is a simple technique that does not account for the statistical properties of the channel and noise, leading to suboptimal channel estimates and higher bit errors. These results highlight the importance of considering the trade-offs between estimation accuracy, computational complexity, and channel knowledge when selecting the appropriate channel estimation scheme for a given communication system and operating conditions.

3.2 Convergence behavior

The channel conditions, described by the various channel estimates have a significant influence on the convergence behavior of the phase shift algorithm. The step size often called the learning rate, is a crucial factor in gradient descent optimization techniques. It specifies the magnitude of the step taken in the negative direction of the gradient throughout each iteration of the optimization process. Step size has a considerable impact on the algorithm's convergence speed and stability. SCA, which entails a gradient descent technique updates the parameters (θ) at each iteration as in the equation below;

$$\theta(t + 1) = \theta(t) - k \nabla f(\theta(t)) \tag{19}$$

Where k indicates the step size, and $\nabla f(\theta(t))$ is the slope of the objective function $f(\theta)$ computed at the current parameter values, $\theta(t)$.

The Armijo algorithm is a line search method that determines the proper step size for each iteration of the gradient descent process. The algorithm determines the biggest step size that meets the Armijo condition, resulting in a lower objective function value at the new point ($\theta(t+1)$) compared to the present position ($\theta(t)$). The Armijo condition is illustrated in the equation below;

$$f(\theta(t + 1) - k \nabla f(\theta(t + 1))) < f(\theta(t) - c k \|\nabla f(\theta(t))\| \tag{20}$$

Where, c is a constant commonly set between 0 and 1, and $\|\nabla f(\theta(t))\|$ is the gradient's norm.

The Armijo algorithm begins with a starting step size k_0 and determines whether the Armijo criterion is met. If the criterion is met, the current step size is acceptable. If not, the step size is decreased by a shrinking factor (usually 0.5 or 0.1) till the requirement is met. The performance of the Armijo algorithm is verified by evaluating the step sizes taken and the number of iterations taken for

the algorithm to converge. These results are indicated in Figure 6 below.

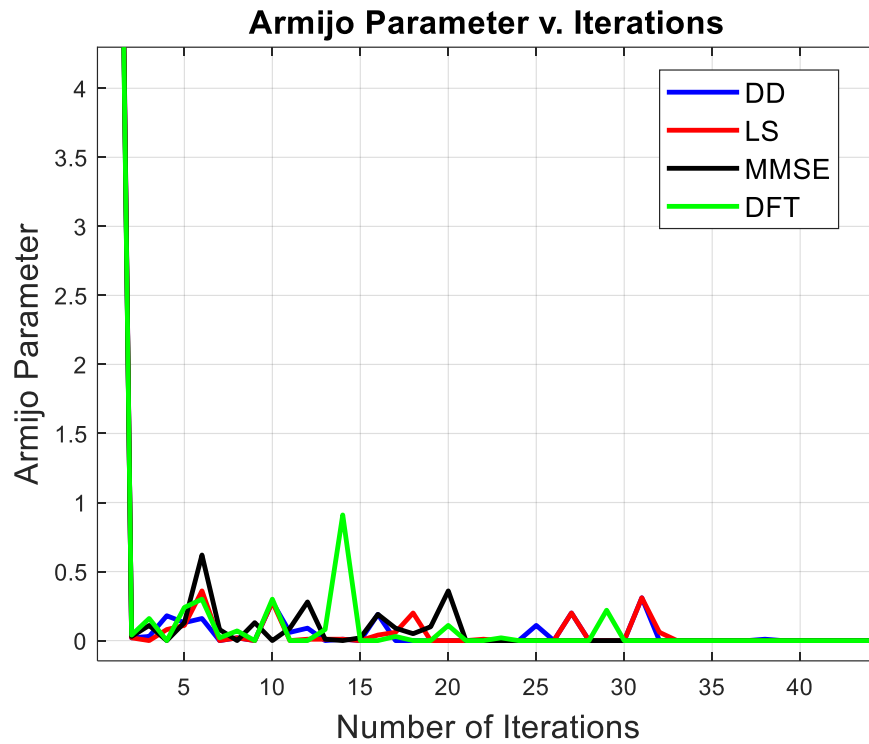


Figure 6. Armijo parameter v. number of iterations

In this work, the starting step size was chosen as 1 and the algorithm iteratively selected a sufficient decrease parameter (the Armijo parameter) for convergence of the SCA algorithm. From the results in Figure 6, the Armijo algorithm shows a good performance since the step size is maintained at 0-1. The DD method shows the fastest convergence at the 20th iteration, followed by the MMSE method at the 25th iteration. Convergence is reached at the 28th iteration with the DFT approach, while the LS shows the poorest performance, with the algorithm converging at the 32nd iteration.

These results indicate that in NLOS scenarios, determining the most appropriate channel estimation method is critical for performance optimization and should be carefully considered to guarantee low BER, high achievable data rate, and minimal complexity. According to the results obtained from this simulation, the DD estimation method appears to be the optimal choice.

4. Conclusion

This work has explored two key aspects of an NLOS communication system; the BER and convergence behavior of the phase shift optimization algorithm following the LS, DD, DFT and MMSE channel estimation methods. An IRS has been proven to significantly improve NLOS communication through passive beamforming. However, acquiring perfect CSI in IRS systems is challenging. Therefore, to leverage the performance gains of an IRS, it is essential to select the best-suited channel estimation method that guarantees the lowest BER and forms the best initialization point for the phase shift algorithm leading to faster convergence at minimal complexity. Results have indicated that while the LS is relatively straightforward, it leads to high BER and causes the phase shift algorithm to converge slowly.

DFT and MMSE offer relatively better performance than LS but it is important to note that MMSE is best suited for cases with the statistical knowledge of the channel characteristics. DD effectively tracks the changes in channel characteristics and updates the channel estimates based on the decoded data symbols, making it more accurate and suitable for NLOS scenarios.

This work was done under the consideration of slow-fading channels. In future, it may be important to evaluate the performance of these algorithms in fast-fading and mobile environments in which case the DD may suffer the disadvantage of relying on outdated initial channel estimates.

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